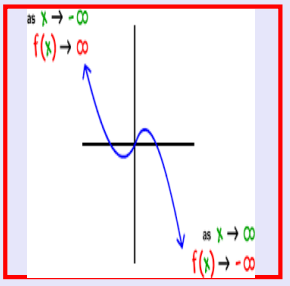
Math 245

Spring 2022

Lecture 12



Solve $x^2 < 3x + 28$ — less than Zero ⇒ − Sign

$x^2 - 3x - 28 < 0$

1) Make RHS = 0.

2) Factor LHS Completely. $(x-7)(x+4) < 0$

3) Solve each Factor by making Factor = 0.

$x - 7 = 0$ $\quad x = 7$ $\qquad$ $x + 4 = 0$ $\quad x = -4$

4) Make the Sign table.

x	$-\infty$	-4	7	∞
$x+4$	−	0	+	+
$x-7$	−	−	0	+
Product	+	−	0	+

$-4 < x < 7$

Graphing: $(-4, 7)$

I.N. $(-4, 7)$ $\qquad$ S.B.N. $\{x \mid -4 < x < 7\}$

Solve $\frac{x^2-8x+15}{x-10} \geq 0$ → greater than 0 ⇒ + Sign

1) RHS = 0, LHS must be one Single Fraction, Completely Factored.

$\frac{(x-5)(x-3)}{x-10} \geq 0$

2) Set each Factor = 0, then Solve.

$x-5=0$, $x-3=0$, $x-10=0$

$x=5$ $x=3$ $x=10$

3) Make Sign table.

x	$-\infty$	3	5	10	∞
$x-3$	−	+	+	+	
$x-5$	−	−	+	+	
$x-10$	−	−	−	+	
Problem	−	+	−	+	

$3 \leq x \leq 5$ $x > 10$

Graphing: [3, 5] ∪ (10, ∞)

I.N. $[3,5] \cup (10,\infty)$

S.B.N. $\{x \mid 3 \leq x \leq 5 \text{ OR } x > 10\}$

Variations

1) Directly

"Multiplication"

ex: y varies directly as x.

$y = kx$

2) Inversely

"Division"

ex: y varies inversely as x^2.

$y = \frac{k}{x^2}$

Constant of Variation.

y varies directly as x. ⇒ $y = kx$

y is 50 when x is 5. ⇒ $50 = k(5)$, $k = 10$

Find y when x is 20.

$y = 10x$

$y = 10(20)$

$y = 200$

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y varies inversely as x^2. ⇒ $y = \frac{k}{x^2}$

y is 5 when x is 2.

$5 = \frac{k}{2^2}$, $5 = \frac{k}{4}$ ⇒ $k = 20$

Find y when x is -2.

$y = \frac{20}{x^2}$

$y = \frac{20}{(-2)^2} = \frac{20}{4} = 5$

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y varies directly as x^3. ⇒ $y = kx^3$

y is 64 when x is 2 ⇒ $64 = k(2)^3$, $64 = k \cdot 8$, $k = 8$

Find y when x is 3.

$y = 8x^3$

$y = 8(3)^3$

$= 8(27)$ ⇒ $y = 216$

y varies inversely as $\sqrt{x}$.

y is 25 when x is 16.

Find y when x is 100.

$$y = \frac{k}{\sqrt{x}}$$

$$25 = \frac{k}{\sqrt{16}}$$

$$25 = \frac{k}{4}$$

$$\boxed{k = 100}$$

$$y = \frac{100}{\sqrt{x}}$$

$$y = \frac{100}{\sqrt{100}} = \frac{100}{10} = 10$$

$$\boxed{y = 10}$$

Factor Completely:

$3x^2 - 5x - 8 = 3x^2 + 3x - 8x - 8$

Product $\Rightarrow -24$

Sum $\Rightarrow -5$

-24: $1,-24$; $2,-12$; $3,-8$; $4,-6$

$= 3x(x+1) - 8(x+1)$

$= \boxed{(x+1)(3x-8)}$

$36x^2 - 49 = (6x)^2 - (7)^2$ $\quad A^2 - B^2$

$= \boxed{(6x+7)(6x-7)}$

$27x^3 + 125 = (3x)^3 + (5)^3$ $\quad A^3 + B^3$

$= \boxed{(3x+5)(9x^2 - 15x + 25)}$

$64x^3 - 27y^3 = (4x)^3 - (3y)^3$ $\quad A^3 - B^3$

$= \boxed{(4x-3y)(16x^2 + 12xy + 9y^2)}$

SG: 5 & Factoring